Problem 1.51

[Computer] Repeat all of Problem 1.50 but using the initial value $\phi_o = \pi/2$.

Solution

Equation (1.51) is on page 31.

$$\ddot{\phi} = -\frac{g}{R}\sin\phi \tag{1.51}$$

With $R = 5$ m and $g = 9.8$ m/s^2 and $\phi_o = \pi/2$, the initial value problem to solve is

$$\ddot{\phi} = -\frac{9.8}{5}\sin\phi, \quad \phi(0) = \frac{\pi}{2},\ \phi'(0) = 0$$

$$\ddot{\phi} = -1.96\sin\phi, \quad \phi(0) = \frac{\pi}{2},\ \phi'(0) = 0.$$

Note that $\phi(0) = \pi/2$ is the angle at $t = 0$, and $\phi'(0) = 0$ indicates that the particle starts from rest. To numerically solve this, type

$$\texttt{s} = \texttt{NDSolve}\left[\left\{\phi''[\texttt{t}] == -1.96\sin[\phi[\texttt{t}]],\ \phi[0] == \frac{\pi}{2},\ \phi'[0] == 0\right\}, \phi, \{\texttt{t}, 0, 21\}\right]$$

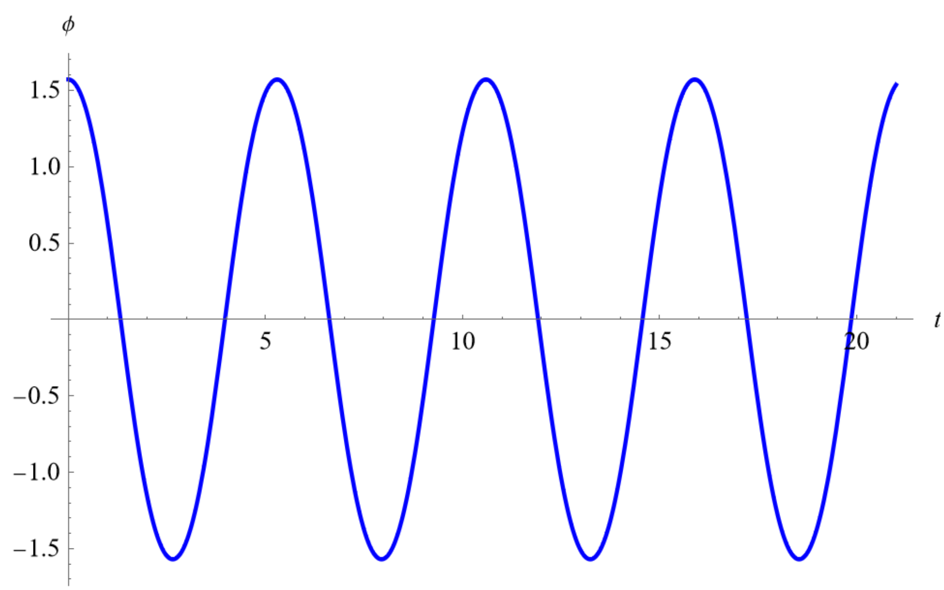
into Mathematica and press `Shift+Enter`. The output below is given as a result.

$$\left\{\left\{\phi \to \texttt{InterpolatingFunction}[\,]\right\}\right\}$$

In order to plot this function, type

$$\texttt{Plot}\left[\texttt{Evaluate}[\phi[\texttt{t}]\ /.\ \texttt{s}],\ \{\texttt{t}, 0, 21\},\ \texttt{PlotRange} \to \texttt{All},\ \texttt{AxesLabel} \to \{\texttt{t}, \phi\},\ \texttt{PlotStyle} \to \texttt{Blue}\right]$$

into Mathematica and press `Shift+Enter` to obtain the following graph.



By making the small-angle approximation, equation (1.51) becomes

$$\ddot{\phi} \approx -\frac{g}{R}\phi,$$

which has the exact solution,

$$\phi(t) = A\cos\left(\sqrt{\frac{g}{R}}\,t\right) + B\sin\left(\sqrt{\frac{g}{R}}\,t\right).$$

Apply the initial conditions to determine the constants, A and B.

$$\phi(0) = A = \frac{\pi}{2}$$

$$\phi'(0) = B\sqrt{\frac{g}{R}} = 0$$

Solving this system of equations yields $A = \pi/2$ and $B = 0$, which means

$$\phi(t) = \frac{\pi}{2}\cos\left(\sqrt{\frac{g}{R}}\,t\right).$$

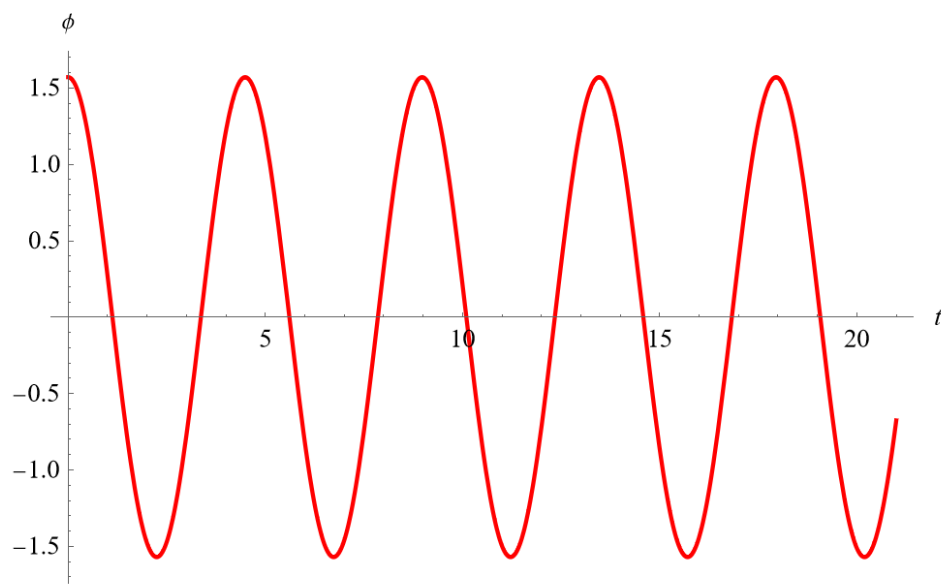
Therefore, with $R = 5$ m and $g = 9.8$ m/s^2,

$$\phi(t) = \frac{\pi}{2}\cos(1.4t).$$

In order to plot this function, type

$$\texttt{Plot}\left[\frac{\pi}{2}\cos[1.4\texttt{t}],\ \{\texttt{t}, 0, 21\},\ \texttt{PlotRange} \to \texttt{All},\ \texttt{AxesLabel} \to \{\texttt{t}, \phi\},\ \texttt{PlotStyle} \to \texttt{Red}\right]$$

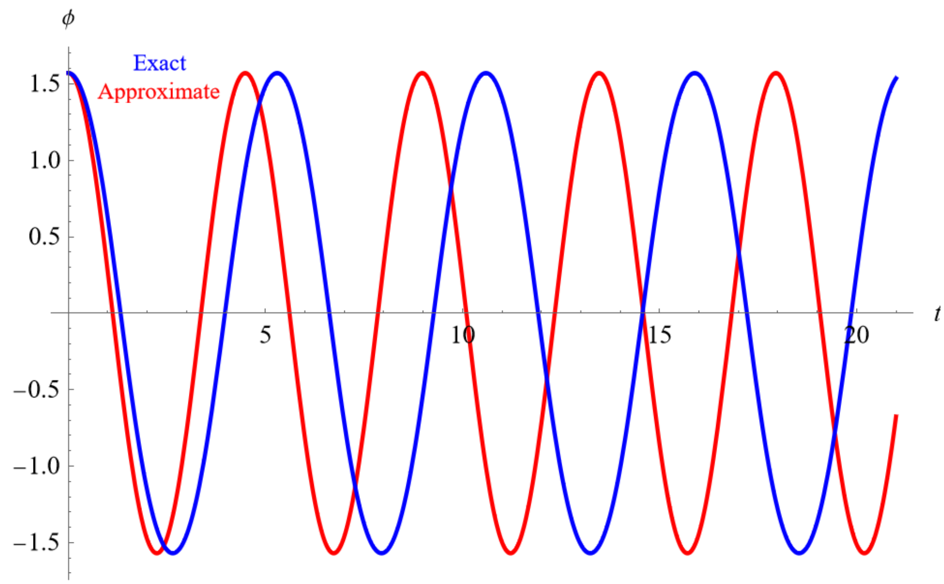
into Mathematica and press `Shift+Enter`.



To superimpose this graph with the previous one, type

$$\texttt{Show}\left[\%, \%\%\right]$$

into Mathematica and press `Shift+Enter`.



Because the graphs do not overlap, the small-angle approximation is not a good one.